



P-value (two sided):

- The p value of the hypothesis testing is the probability of obtaining a value of the test statistic as extreme or more extreme (in the direction of supporting H_A) than the one actually computed.
- In a two-tailed test, we are interested in extreme values on both ends of the distribution (positive and negative Z-values). For a Z-test:
 - Calculate the probability of getting a value greater than or equal to the observed Z-sample on the positive side: P ($Z \ge Z$ sample).
 - Add this probability to the corresponding probability on the negative side: P ($Z \le -Z$ sample)

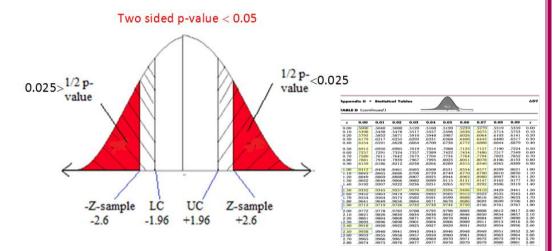
★ Example:

Tablets of a drug are compressed in a machine at target average weight 300 mg. A batch was made and a random sample of 30 tablets showed an average weight of 307. The company has specified σ to be 15 mg. Does the batch meet target tablet weight at significance level (α) of 0.05.

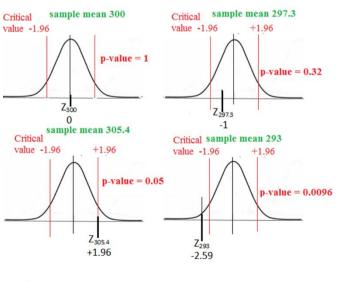
✓ The p value indicates the probability of observing $Z \ge Z$ -sample or $Z \le -Z$ -sample.

✓ In our case study: $P(Z \ge 2.6) = 0.0047$ (this is the area in the right tail beyond Z=2.6). $P(Z \le -2.6) = 0.0047$ (this is the area in the left tail beyond Z=-2.6).

✓ Since the test is two-tailed: p-value = $P(Z \ge 2.6) + P(Z \le -2.6) = 0.0047 + 0.0047$ = 2×0.0047 = 0.0094



- > If in the case study, 3 more batches were made and tested at the same sample size and α (0.05), but they had different sample means of 300, 297.3, 305.4 and 293. Calculate the p-value for each batch.
- When the Z score of the sample mean is within the critical region (-1.95 to +1.96), p-value is 0.05 or higher (Fail to reject H₀ as test statistic is within the acceptance region). When it is outside the critical region p-value is less than 0.05 (reject the H₀ as test statistic is in the rejection region). At $\alpha = 0.05$, the best case scenario and the worst case scenario for the acceptance of H₀ are having Z scores of sample mean as 0 and ±1.96, respectively.
- As Z-sample higher deviates positively from the upper critical value or negatively from the lower critical value, p-value becomes smaller than 0.05 and thus the evidence against H₀ becomes stronger.



۶

• Conventions for interpreting *P* values at $\alpha = 0.05$ (most commonly used)

- ✓ In statistical hypothesis testing, p-values are used to determine the level of significance of results. The levels of significance help us decide whether to reject or fail to reject the null hypothesis (H₀)
- ✓ P ≥ 0.05 Result is not significant: usually indicated by no asterisk.
- ✓ P < 0.05 Result is significant: usually indicated by one asterisk (*)
- ✓ P < 0.01 Result is highly significant: usually indicated by two asterisks (**)
- ✓ P < 0.001 Result is very highly significant: usually indicated by three asterisks (***)
- > If for the case study, 3 more batches were made and tested at the same sample size and α (0.05), but they had different sample means (SM) of 294, 292 and 290. The three batches showed different significant differences from the hypothesized population mean as indicated by asterisks for different p-values.
- At the same significance level, z-critical values are the same (±1.96 for α=0.05), but samples of different means have different Z-scores (Z-sample) and as these scores are more different from the critical values toward the tails, lower p-values are obtained.

Batch	SM	SE	Z-sample	P-two sided
1	294	2.7	-2.22	0.0264*
2	292	2.7	-2.96	0.003**
3	290	2.7	-3.7	0.0002***

• Making a Decision:

- Decision Rule Based on P-value
- > To use a *P*-value to make a conclusion in a hypothesis test, compare the *P*-value with α .
 - ✓ If $P < \alpha$, then reject H_0 .
 - ✓ If $P > \alpha$, then fail to reject H_0 .

		IM
Decision	Claim is H ₀	Claim is H _a
Reject H ₀	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Do not reject H _o	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

• Testing H₀ by Means of a Confidence Interval

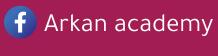
- > When testing a null hypothesis by the mean of confidence interval, we reject H_0 at the α level of significance (0.05 in the case study) if the hypothesized mean (suggested by H0) is not contained within the 95% confidence interval of the sample mean.
- > If the hypothesized mean is contained within the interval, H_0 cannot be rejected at α level of significance.
- For the case study: The hypothesized parameter (300) is not contained in the confidence interval of the sample mean. Thus, the null hypothesis is rejected.
- \blacktriangleright The hypothesized mean < LL so the sample came from the upper rejection region.

$$\overline{X} \pm Z_{1-\alpha/2} * \frac{\sigma}{\sqrt{n}} =$$

$$307 \pm 1.96 \times 2.7$$

$$301.7 to 312.3$$





O Arkanacademy

🛞 www.arkan-academy.com

+962 790408805